MATH 141: Quiz 3

Name: key

Directions:

- Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. Find whether the following limits are a number, $\pm\infty$, or does not exist:

(a) $\lim_{x\to 3} x-3 = 3-3 = \boxed{0}$

(b) $\lim_{t\to -\frac{17\pi}{6}} \sin(t)\cdot\cos(t) = \sin\left(-\frac{17\pi}{6}\right)\cdot\cos\left(-\frac{17\pi}{6}\right)$

$= -\frac{1}{2}\cdot\left(-\frac{\sqrt{3}}{2}\right)$

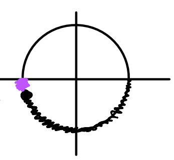
$= \boxed{\dfrac{\sqrt{3}}{4}}$

$t = -\frac{17\pi}{6}$

$= -\frac{12\pi}{6} - \frac{5\pi}{6}$

ignore

$= \underline{-2\pi} - \frac{5\pi}{6}$



$\bar{t} = \frac{\pi}{6}$

so terminal point is $P\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

(c) $\lim_{x\to 2} \dfrac{\sqrt{4x+1}-3}{x-2}$

limit laws give $\frac{0}{0}$:

$$= \frac{\sqrt{4\cdot 2+1}-3}{2-2} = \frac{\sqrt{9}-3}{0} = \frac{0}{0}$$

$\lim_{x\to 2}$ tells you create a global factor of $(x-2)$. Rationalize numerator.

$$\lim_{x\to 2} \frac{\overbrace{\sqrt{4x+1}}^{A} - \overbrace{3}^{B}}{x-2}\cdot\frac{\overbrace{\sqrt{4x+1}}^{A} + \overbrace{3}^{B}}{\sqrt{4x+1}+3} = \lim_{x\to 2}\frac{\left(\sqrt{4x+1}\right)^2 - 3^2}{(x-2)\left(\sqrt{4x+1}+3\right)}$$

$$= \lim_{x\to 2}\frac{4x+1-9}{(x-2)\left(\sqrt{4x+1}+3\right)}$$

$$= \lim_{x\to 2}\frac{4x-8}{(x-2)\left(\sqrt{4x+1}+3\right)}$$

$$= \lim_{x\to 2}\frac{4\cancel{(x-2)}}{\cancel{(x-2)}\left(\sqrt{4x+1}+3\right)}$$

$$= \lim_{x\to 2}\frac{4}{\sqrt{4x+1}+3}$$

$$= \frac{4}{\sqrt{4\cdot 2+1}+3} = \frac{4}{3+3} = \boxed{\frac{2}{3}}$$

2. Given

$$f(x) = \begin{cases} x^2 & x \leq 1 \\ x - 1 & x > 1 \end{cases}$$

use the **three-part definition of continuity** to determine whether $f(x)$ is continuous at $x = 1$.

Not using the three part definition results in zero credit.

1 $f(1) = 1^2 = 1$ defined

2 $\lim_{x \to 1} f(x)$ DNE because

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1^2 = 1$ while

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x - 1 = 1 - 1 = 0$

Since one condition of continuity was violated, $f(x)$ is not continuous at $x = 1$. No need to keep checking.